Intensity correlations in resonance nonlinear magneto-optical rotation

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We have studied the intensity correlations between two orthogonally linearly polarized components of a laser field propagating through a resonant atomic medium. These experiments have been performed in a Rubidium atomic vapor. We observe that the correlations between the orthogonally polarized components of the laser beam are maximal in the absence of a magnetic field. The magnitude of the correlations depends on the applied magnetic field, and the magnitude first decreases and then increases with increasing magnetic field. Minimal correlations and maximal rotation angles are observed at the same magnetic fields. The width of the correlation function is directly proportional to the excited state lifetime and inversely proportional to the Rabi frequency of laser field. These results can be useful for improving optical magnetometers and for optical field or atomic spin squeezing.

I. INTRODUCTION

Quantum coherence effects have been a focus of research activities for the last two decades, because they may drastically change the optical properties of a medium. For example, electromagnetically induced transparency (EIT) [1, 2], predicted and observed in CW and pulsed regimes [3, 4], practically allows absorption to vanish. The nonlinear response of a resonant atomic medium at moderate optical intensities can be strongly enhanced by creating maximal coherence between long-lived sublevels of the ground state [2, 5, 6, 7, 8]. The corresponding steep dispersion results in the ultra-slow or fast propagation of light pulses [9, 10, 11], which can produce huge optical delays [12] and can be used for drastic modification of the phase-matching conditions for Brillouin scattering [13], and four-wave mixing [14]. It is possible to manipulate a coherent medium and produce optical pulses at rates faster than the relaxation rates of the medium [15], in order to develop bright sources for efficient generation of IR and FIR pulses [16]. Also, the nonlinear properties of such media are enhanced, allowing the implementation of quantum light storage [17, 18], nonlinear optics at a few photon level [19], and other nonlinear effects [20].

It is worth noting here that media with quantum coherence might be used for the sensitive measurement of magnetic fields [21]. Recent interest in nonlinear magneto-optical effects (NMOE), such as nonlinear magneto-optical rotation (NMOR) [5] or nonlinear polarization self-rotation (NPSR) [22], has been fueled by possible applications to precision magnetometry [23, 24, 25, 26].

The amplitude noise of optical fields increases after the fields interact with an atomic media (atomic excess noise) [27, 28, 29, 30, 31]. Several processes, such as the conversion of phase noise to amplitude noise and the four-wave mixing process, can be associated with the generation of atomic noise [27, 28, 29, 30, 31, 32]. The strong coupling of optical fields in Λ-type atoms provides a way of controlling amplitude fluctuations. The amplitude correlations have been studied in EIT [33, 34, 35, 36, 37] and NMOE [38] experiments. Observation of sub-Poisson statistics of the amplitude noise is reported in [34]. Amplitude correlations and anti-correlations are observed in two orthogonally circularly polarized optical beams from the same laser [36] and in orthogonally linearly polarized optical beams from two independent lasers [35]. Power spectra of intensity fluctuations was studied in [37]. Noise spectroscopy of nonlinear magneto-optical resonances in Rb vapor at a detection frequency 2.5 MHz with bandwidth 30 kHz is performed in [38], where the photon noise limit has been reached.

Coherent effects may be used to reduce the noise level below the limit given by photon fluctuations. Several schemes for squeezing of the fields are proposed and studied in EIT [39, 40, 41] or NPSR [32, 42, 43, 44, 45] configurations. It has been demonstrated that the atomic excess noise can reduce squeezing in the fields [45], and the addition of excess noise to quantum state of radiation is important and should be taken into account in experiments involving coherently prepared atoms. For instance, recently a considerable influence of the excess noise on light storage has been observed [46].

In this paper, we have considered the atomic noise correlations in an NMOR experiment with a rubidium vapor. In a previous paper [36], we studied the fluctuations between left and right circularly polarized beams, which are the normal modes of the system. Here, we have studied the fluctuations between two orthogonally linearly polarized beams, which are not the normal modes of the system. Our current experimental configuration is widely used in optical magnetometry measurements [5],

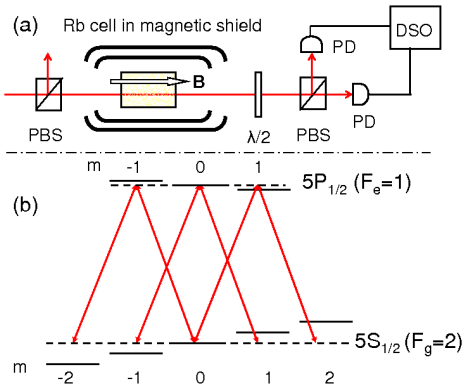

FIG. 1: Experimental setup (a) and energy levels (b). The ground state Zeeman splitting of Rb^{87} is 0.7 MHz/G.

and therefore furnishes an ideal system for studying these fluctuations. We have observed the variation of the correlation function $G^{(2)}(\tau)$ (τ is the delay time) as a function of the magnitude of an applied magnetic field.

The paper is organized as follows. In Section II, we describe the experimental setup and the results. In Section III, we present a theoretical model, which gives us an explanation of the results. Finally, in Section IV, we present our conclusions.

II. EXPERIMENTAL SETUP AND OBTAINED RESULTS

The schematic of our experimental setup is shown in Fig. 1a. An external cavity diode laser [47] (the linewidth is less than 1 MHz) is tuned to the center of the Doppler broadened D_1 line (transition $5S_{1/2}(F=2) \rightarrow 5P_{1/2}(F=1)$) of ^{87}Rb (see Fig. 1b). After a polarizing beam splitter (PBS), the laser beam (of diameter 0.1 cm and power 0.9 mW) enters a glass cell (of length 7.5 cm) filled with an atomic Rubidium vapor (with natural isotope abundance and atomic density 10^{12} cm^{-3}).

The cell is placed inside a two-layer magnetic shield that protects the cell from uncontrolled environmental magnetic fields. A solenoid is installed inside the shield to create a longitudinal magnetic field B. A half-wave plate is placed between the two PBSs to rotate the polarization of the output beam to 45^o with respect to transmission axis of the second PBS. This angle is set by making the two beams have equal intensities at a large one-photon detuning from the atomic resonance in the cell at room temperature. Two orthogonally linearly polarized beams from the second PBS are focused on identical fast photodetectors (PDs) with a frequency bandwidth of 75 kHz-1.2 GHz. The optical path lengths for both beams and length of cables between the PDs and the data recorder are chosen to be the same to avoid a time delay between the signals in two registration channels. The signals from PDs are recorded by a digital storage oscilloscope (DSO) with a bandwidth of 100 MHz.

We have also studied the transmission and the polarization rotation of the beams as a function of the two-photon detuning by scanning the magnitude of the longitudinal magnetic field B at a frequency of several Hz. For this, the fast photo-detectors have been replaced by low-frequency detectors to monitor the output from the second beam splitter. Then, using the recorded signals, S_1 and S_2, we calculate the transmission by

$$T = \left(\frac{S_1 + S_2}{S_{01} + S_{02}} \right), \qquad (1)$$

and the polarization rotation by

$$\phi = \arcsin \left(\frac{S_1 - S_2}{S_1 + S_2} \right), \qquad (2)$$

where S_{01} and S_{02} are the signals from the photodetectors without atomic resonance absorption.

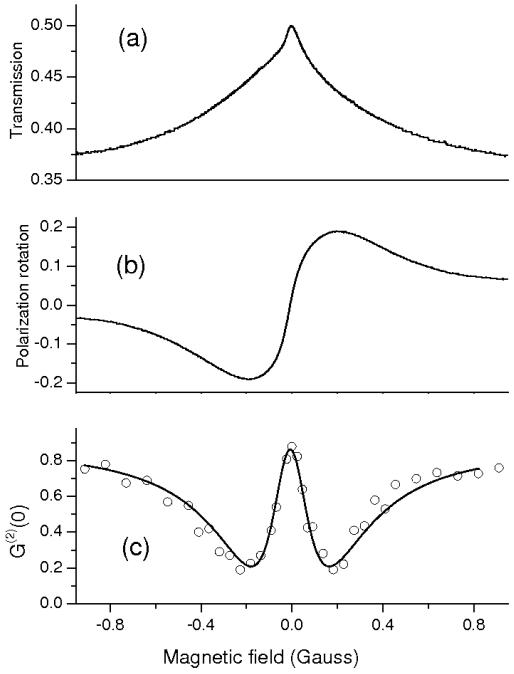

FIG. 2: Magnetic field dependence of the EIT (Fig. 2a), polarization rotation (Fig. 2b) and correlation function $G^{(2)}(0)$) (Fig. 2b). In Fig. 2(c) the circles represent experimental data and the solid curve is a visual guide for eyes.

We start the presentation of the results with a demonstration of the narrow EIT resonance. The narrow EIT resonance is related to the "dark state" of Rb atoms formed by the action of two laser fields. The transmission T versus magnetic field B is shown in Fig. 2a. In a magnetic field, the linear polarization of the laser field rotates, and the polarization rotation dependence on the external magnetic field is shown in Fig. 2b. The width of nonlinear magnetic optical resonances is determined by the power broadening of the two-photon transition, which for our conditions is narrower than the natural optical width (6 MHz). The observed background in Fig. 2b is due to the linear rotation of polarization.

To study the fluctuations of an optical field transmitted through a dense Rb vapor, we have registered the time dependent signal fluctuations $\delta S_{1,2}(t)$ of the two optical beams after the second PBS. We record these signal fluctuations for the two output beams in a 10 μs time-window for different magnetic fields, and then we calculate the normalized correlation functions given by

$$G^{(2)}(\tau) = \frac{\langle \delta S_1(t) \delta S_2(t+\tau) \rangle}{\sqrt{\langle [\delta S_1(t)]^2 \rangle \langle [\delta S_2(t+\tau)]^2 \rangle}}, \qquad (3)$$

where $\delta S_{1,2}(t)$ are the time dependent fluctuations of the two beams, and stochastic averaging [51] denoted by angular brackets is defined as $\langle Q(t) \rangle \equiv 1/T \int_t^{t+T} Q(t')dt'$.

We are unable to detect any intensity fluctuations when the cell has been removed from the laser beams. Of course, it is well-known that diode laser radiation possesses low intensity noise. In our experiments, the photo-detector noise dominates, as in Refs. [36, 37].

In the presence of a resonant medium, however, the situation changes, and the phase noise of a diode laser is transformed into intensity fluctuations. The magnetic dependence of the correlation function at zero time delay, $G^{(2)}(0)$, is shown in Fig. 2c. As one can see, a correlation magnitude of close to 0.9 is obtained at zero magnetic field.

Increasing the magnetic field up to 0.2 G results in decreasing the correlation function magnitude to ~ 0.2. But further increase of the magnetic field leads to a revival of the correlation, and it reaches ~ 0.7 at 0.8 G field strength. We underline here that this behavior is different from that observed in [36]. The key difference is that, in this paper, we study the fluctuations of the orthogonal linear polarization components, which are not normal modes because of the Faraday effect.

By examining the polarization angle rotation dependence shown in Fig. 2b, one can conclude that the maximum correlation is obtained when the magnetic field and the polarization rotation in the atomic medium are close to zero. The correlation functions $G^{(2)}(\tau)$ for three different magnetic fields are shown in Fig. 3.

The signals from the photo-detectors in time intervals of 200 ns are shown in the inset boxes in Fig. 3a,b,c. The signals are proportional to the laser beam power with a slope of 500 V/W. The vertical range of the plot is 4 mV.

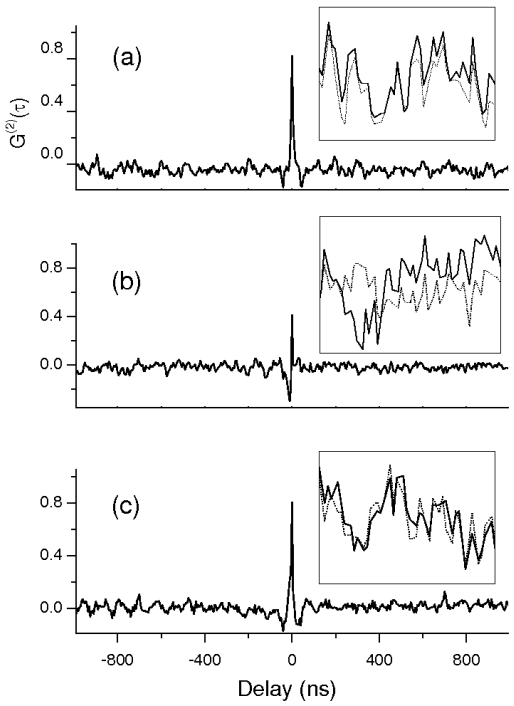

FIG. 3: Intensity correlation function $G^{(2)}(\tau)$: curve (a) recorded at zero magnetic field, curve (b) at a magnetic field of 0.18 G and curve (c) at a magnetic field of 0.9 G. Also signals from photo-detectors are shown in boxes. The waveforms are recorded in time intervals of 200 ns and amplitude intervals of 4 mW.

The amplitude variations of the signals are practically the same as in Fig. 3.

The temporal behavior of the signal is modified by the applied magnetic field. The signals shown in Fig. 3a and 3c are correlated, and the ones shown in Fig. 3b (box) are not. We observe that at low and high magnetic field, fluctuations are strongly correlated. At an intermediate magnetic fields, the peaks of correlation function have less magnitude. The correlation function behaves as if it changes sign around zero time delay. Possibly different frequency components of the signals can be correlated or anti-correlated at these magnetic fields. For the dependence shown in Fig. 2c, we have selected the magnitudes of the correlation function at zero time delay.

We note that the inverse width of the peaks is the order of $2\pi \cdot 20$ MHz, and it is comparable with the width of the saturation resonance [48, 49, 50], and it is much broader than the ground state relaxation rate. The inverse width depends on the excited state decay rate $2\pi \cdot 6$ MHz and the optical excitation rate. The influence of one-photon optical saturation on MNOR is discussed in [5].

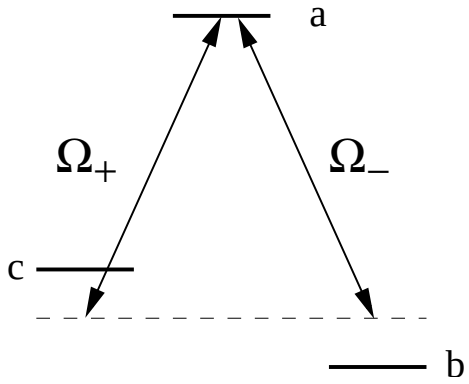

FIG. 4: A simplified three-level scheme. Splitting of the ground state is $\omega_{cb} = 2g\mu_B B$, where g is the Lande factor, μ_B is Borh's magneton, and B is the magnetic field.

III. THEORETICAL APPROACH AND DISCUSSION

The following is a simplified theoretical description of our system treating the laser fields classically and using a density matrix for the atomic response. The laser beams are in resonance with a three-level medium as depicted in Fig. 4.

The Hamiltonian of the atom is given by

$$\hat{H} = \hbar\Omega_-|a\rangle\langle b| + \hbar\Omega_+|a\rangle\langle c| + h.c., \tag{4}$$

where $\Omega_\pm = \wp_{c,b}E_\pm/\hbar$ are the Rabi frequencies of left- and right-circularly polarized beams; $\wp_{c,b}$ and $E_\pm$ are the corresponding dipole moments of the atomic transitions and the electric fields. The density matrix equation is given by

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{2}(\hat{\Gamma}\rho + \rho\hat{\Gamma}), \tag{5}$$

where $\hat{\Gamma}$ is the relaxation matrix, introduced to describe relaxation processes in the atomic medium [51]. The equations for field propagation are

$$\frac{\partial\Omega_-}{\partial z} = -i\eta_b\rho_{ab}, \quad \frac{\partial\Omega_+}{\partial z} = -i\eta_c\rho_{ac}, \tag{6}$$

where $\eta_b = \nu_- N\wp_b^2/(2\hbar\epsilon_0 c)$, $\eta_c = \nu_+ N\wp_c^2/(2\hbar\epsilon_0 c)$ are the coupling constants, $\nu_\pm$ are the frequencies of circular polarized fields, $\wp_{b,c}$ are the dipole moments of the corresponding transitions, N is the density of medium. Note that the left- and right-polarized beams are the normal modes of the current system, and their polarizations do not change while they are propagating through the cell.

The intensities of the optical beams propagating through the cell fluctuate. The corresponding correlation function is defined as

$$G^{(2)}(\tau) = \frac{\langle\delta I_-(t)\delta I_+(t+\tau)\rangle}{\sqrt{\langle[\delta I_-(t)]^2\rangle\langle[\delta I_+(t+\tau)]^2\rangle}}. \tag{7}$$

Intensity fluctuations have been experimentally and theoretically studied in [33, 36], and they are related to the atomic responses

$$\delta I_- \sim \Im\rho_{ab}\Omega_-^*, \quad \delta I_+ \sim \Im\rho_{ac}\Omega_+^*. \tag{8}$$

The corresponding atomic coherences in the three-level system can be found from the solution of the density matrix equations. Assuming the phase diffusion of optical fields is a slow process, the atomic coherences are given by

$$\rho_{ab} = -i\frac{n_{ba}\Omega_- + \rho_{cb}\Omega_+}{\Gamma_{ab}}, \quad \rho_{ca} = i\frac{n_{ca}\Omega_+ + \rho_{cb}\Omega_-}{\Gamma_{ca}} \tag{9}$$

$$\Gamma_{cb}\rho_{cb} = i\rho_{ca}\Omega_- - i\rho_{ab}\Omega_+, \tag{10}$$

where $\Gamma_{ab} = \gamma_{ab} + i(\omega_{ab} - \nu_-)$; $\Gamma_{ca} = \gamma_{ca} - i(\omega_{ac} - \nu_+)$; $\Gamma_{cb} = \gamma_{cb} + i(\omega_{cb} - \nu_- + \nu_+)$; $n_\alpha = \rho_{\alpha\alpha}$; $n_{\alpha\beta} = \rho_{\alpha\alpha} - \rho_{\beta\beta}$; $\omega_{\alpha\beta}$ are the atomic frequencies; α and β are labels for atomic levels a, b, c; $\nu_-(t) = \nu_+(t)$ are the instantaneous frequencies of laser radiation in both beams having orthogonal polarizations. Assuming $\Omega_- = \Omega_+ = \Omega$, and defining $\delta = \omega_{ab} - \nu_- - \omega_{cb}/2$, we solve Eq. (10) with respect to ρ_{cb}. Then, substituting the solution into Eqs. (9) gives us the following:

$$\rho_{ab} = i\frac{\Gamma_{cb}(\Gamma - i\delta)n_{ab} + n_{cb}\Omega^2}{\Gamma_{cb}(\delta^2 + \Gamma^2) + 2\Gamma\Omega^2}\Omega, \tag{11}$$

$$\rho_{ca} = i\frac{\Gamma_{cb}(\Gamma + i\delta)n_{ca} + n_{cb}\Omega^2}{\Gamma_{cb}(\delta^2 + \Gamma^2) + 2\Gamma\Omega^2}\Omega. \tag{12}$$

The set of equations for populations can be obtained by substituting Eqs.(11,12) into Eq.(5) for the appropriate atomic populations:

$$\gamma_a n_a + \left(\frac{\Gamma_{cb}(\Gamma - i\delta)}{\tilde{\Gamma}_{cb}} + \frac{\Gamma_{bc}(\Gamma^* + i\delta)}{\tilde{\Gamma}_{cb}^*}\right)\Omega^2 n_{ab} + \left(\frac{1}{\tilde{\Gamma}_{cb}} + \frac{1}{\tilde{\Gamma}_{cb}^*}\right)\Omega^4 n_{cb} = 0, \tag{13}$$

$$\gamma_a n_a + \left(\frac{\Gamma_{cb}(\Gamma + i\delta)}{\tilde{\Gamma}_{cb}} + \frac{\Gamma_{bc}(\Gamma^* - i\delta)}{\tilde{\Gamma}_{cb}^*}\right)\Omega^2 n_{ac} + \left(\frac{1}{\tilde{\Gamma}_{cb}} + \frac{1}{\tilde{\Gamma}_{cb}^*}\right)\Omega^4 n_{bc} = 0, \tag{14}$$

where $\tilde{\Gamma}_{cb} = \Gamma_{cb}(\delta^2 + \Gamma^2) + 2\Gamma\Omega^2$. By introducing $A_b = A - \delta B$, $A_c = A + \delta B$, and

$$A = \left(\frac{\Gamma_{cb}\Gamma}{\tilde{\Gamma}_{cb}} + \frac{\Gamma_{bc}\Gamma^*}{\tilde{\Gamma}^*_{cb}} \right) \Omega^2, \quad B = -i \left(\frac{\Gamma_{cb}}{\tilde{\Gamma}_{cb}} - \frac{\Gamma_{bc}}{\tilde{\Gamma}^*_{cb}} \right) \Omega^2, \tag{15}$$

$$C = \left(\frac{1}{\tilde{\Gamma}_{cb}} + \frac{1}{\tilde{\Gamma}^*_{cb}} \right) \Omega^4, \tag{16}$$

the set equations for the populations can be re-written in the compact form

$$\gamma_a n_a + A_b n_{ab} + C n_{cb} = 0, \tag{17}$$

$$\gamma_a n_a + A_c n_{ac} + C n_{bc} = 0, \tag{18}$$

and solved (see Appendix A). The solution for the population difference in level b and c is given by

$$n_{cb} = \frac{2\gamma_a B\delta}{3A_bA_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)}. \tag{19}$$

Simplifying Eqs. (15,16), we obtain $A \simeq \gamma_{cb}$, $B \simeq \frac{\Delta}{\gamma}$, $C \simeq \frac{|\Omega|^2}{\gamma}$, and

$$n_{cb} \simeq \frac{\Delta\delta}{|\Omega|^2}. \tag{20}$$

The intensity fluctuations are determined by atomic coherences, which are

$$\Im\rho_{ab} \simeq \frac{\gamma_{cb}\gamma + \Delta\delta}{\gamma|\Omega|}, \quad \Im\rho_{ac} \simeq \frac{\gamma_{cb}\gamma - \Delta\delta}{\gamma|\Omega|}, \tag{21}$$

where the field phase fluctuation is related to the frequency deviation δ that is restricted by the EIT window [36], $\delta \sim \frac{\Omega^2}{\gamma}$.

We can clearly see from Eq.(21) that the intensity fluctuations have two contributions: the first originates from absorption, which is the same for both modes, and the second originates from the Raman term appearing from the population difference n_{cb}, which has opposite signs for these two modes. One field is amplified due to absorption of the second field. Depending on the one- and two-photon detuning, Δ, the intensity fluctuations occur in phase if $\gamma_{cb}\gamma^2/\Omega^2 \gg \Delta$ or out of phase if $\gamma_{cb}\gamma^2/\Omega^2 \ll \Delta$. The last condition gives rise to the anti-correlations.

Let us note here that at the EIT condition, $\Delta = 0$, correlated intensities can be also considered from the point of view of matched pulses [52]. The details of a study of switching between correlations and anti-correlations can be found in [33, 36]

Now we are ready to consider intensity fluctuations between the orthogonally linearly polarized beams in the current experiment. Linearly polarized light can be regarded as a linear combination of left- and right-circular polarized light. The light with left- and right-circular polarizations do not change their state of polarization while propagating through the gas cell. We previously studied the correlation of intention fluctuations between the normal modes [36].

In the current experiment, to detect rotation of polarization, we use the following scheme. After the polarizer, the beam propagates through the cell, and then after a half-wave plate, the polarization plane rotates 45^o degrees and is split by a PBS into two orthogonally polarized optical beams. The intensity of each beam is detected. Thus, the measured signals, S_1 and S_2, are proportional to the intensities of the propagated circularly polarized components, which are given by

$$S_{1,2} \sim I_{1,2} = \frac{1}{2}(I_+ + I_- \pm 2\sqrt{I_-I_+}\sin\phi), \tag{22}$$

and the polarization rotation is given by

$$\phi = \arcsin\left(\frac{I_1 - I_2}{I_1 + I_2} \right). \tag{23}$$

Then correlation between S_1 and S_2 can be calculated by

$$G^{(2)}(\tau) = \frac{\langle \delta I_1(t)\delta I_2(t+\tau)\rangle}{\sqrt{\langle[\delta I_1(t)]^2\rangle\langle[\delta I_2(t+\tau)]^2\rangle}}. \tag{24}$$

As is clearly seen from Eq.(22) and Eq.(24), the circularly polarized beams are equally split into two linear polarizations. Thus, the two intensities of the linearly polarized components are correlated. The only contribution that decreases the correlations is due to the terms depending on the rotational angle, which have different signs for orthogonal polarizations in Eq.(22).

The intensity fluctuations should be small for the components that are not perfectly anti-correlated. But as we have shown in a previous paper [36], increasing the magnetic field causes these components to become anti-correlated. On another hand, increasing the magnetic field increases the rotation at first, but then once the splitting becomes bigger than the EIT width, the rotation angle decreases, restoring the correlations between the intensities of the linear components.

One can see from Eq.(24) that if the rotational angle in a magnetic field is small, the beams mainly consist of the sum of intensities for left- and right-circularly polarized beams, and thus the fluctuations are correlated. Anti-correlations come from the terms having different signs due to Faraday rotation in the magnetic field. So, at zero magnetic field, we have only correlations. Note here that the sum of intensities is a major contribution and always contributes to correlations between intensities. But for stronger magnetic fields, due to the decreasing of the angle of rotation, this term vanishes, and only correlation between beams is observed.

In the experiment, on the other hand, we observe the decreasing of correlation with increasing magnetic field

intensity. It looks similar to the results obtained for circular polarization, but the physics is completely different. Actually, the strong decrease of correlations observed in the experiment occurs because of strong anti-correlations for higher two-photon detunings.

For larger two-photon detuning, the nonlinear magneto-optical rotation of polarization becomes smaller and intensity correlations restore their correlations. Define

$$I_+ = I_0 + i_+, \quad I_- = I_0 + i_-, \tag{25}$$

where $i_\pm$ are the intensity fluctuations of the beams $I_\pm$, correspondingly. Then, using $\sqrt{1+z} \simeq 1 + \frac{z}{2} - \frac{z^2}{8}$, introducing $x = i_+ + i_-$ and $s = i_+ - i_-$ (note that $i_+ i_- - \frac{x^2}{4} = 4i_+ i_- - (i_+ + i_-)^2 = -(i_+ - i_-)^2 = -s^2$), we can rewrite Eqs. (22) as

$$I_{1,2} = 2I_0 + x \pm (2I_o + x - \frac{s^2}{4I_0}) \sin\phi. \tag{26}$$

Thus we obtain

$$\delta I_{1,2} = I_{1,2} - \langle I_{1,2}\rangle = x(1 \pm \sin\phi) \mp \frac{s^2 - \langle s^2\rangle}{4I_0} \sin\phi \tag{27}$$

and

$$\langle (I_1 - \langle I_1\rangle)^2\rangle = \langle x^2\rangle (1+\sin\phi)^2 + \frac{\langle s^4\rangle - \langle s^2\rangle^2}{16 I_0^2} \sin^2\phi \tag{28}$$

Finally, we can calculate correlation function $G^{(2)}$, defined by Eq.(24),

$$G^{(2)}(\tau) = \frac{\langle x^2\rangle \cos^2\phi + \frac{\langle s^4\rangle - \langle s^2\rangle^2}{16 I_0^2} \sin^2\phi}{\sqrt{\left(\langle x^2\rangle \cos^2\phi + \left(\frac{\langle s^4\rangle - \langle s^2\rangle^2}{16 I_0^2}\right)\sin^2\phi\right)^2 + 4\langle x^2\rangle \frac{\langle s^4\rangle - \langle s^2\rangle^2}{16 I_0^2} \sin^4\phi}} \tag{29}$$

In the last equation, one can see that for small magnetic fields, when the rotation angle is small, and the beam intensities are correlated [36], and for strong magnetic fields, when the rotation angle is also small and the beam intensities are anti-correlated [36], the correlation function equals to unity. These results are independent of correlations or anti-correlations between the beams. Correspondingly, in the intermediate case, we have correlations that are less perfect.

The foregoing discussion furnishes a qualitative description of the correlation behavior we have observed in this experiment. To obtain quantitative agreement, we would need to take into account the intensity fluctuations more accurately, beyond the small variations as we have assumed here. We will present the results of such an analysis elsewhere.

IV. CONCLUSIONS

We have studied atomic noise correlations in a nonlinear magneto-optical rotation experiment with Rubidium atomic vapor by using broadband detection. The correlations between the orthogonally polarized components of the laser beam are maximal in the absence of a magnetic field. The width of the correlation function peak is proportional to the excited state lifetime and the inverse Rabi frequency. When a longitudinal magnetic field is applied, the correlations first decrease and then increase. The minimal correlations and the maximal rotation angles are observed at the same magnetic fields. These results can be useful for improving optical magnetometers and for squeezing optical fields or atomic spins.

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APPENDIX A: SOLUTION OF SET OF THE DENSITY MATRIX EQUATIONS

The set of equations from populations Eq.(17,18) is given by

$$\gamma_a n_a + A_b n_{ab} + C n_{cb} = 0 \tag{A1}$$

$$\gamma_a n_a + A_c n_{ac} + C n_{bc} = 0 \tag{A2}$$

where A_b, A_c, C are defined above (see Eqs.(15,16)). Then solution of Eqs.(A1,A2) are the following

$$n_a = \frac{A_b A_c + C(A_b + A_c)}{3A_b A_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)} \tag{A3}$$

$$n_b = \frac{A_b A_c + C(A_b + A_c + 2\gamma_a) + A_c\gamma_a}{3A_b A_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)} \tag{A4}$$

$$n_c = \frac{A_b A_c + C(A_b + A_c + 2\gamma_a) + A_b\gamma_a}{3A_b A_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)} \tag{A5}$$

and population differences can be presented in the following form

$$n_{ab} = -\frac{\gamma_a(A_c + 2C)}{3A_b A_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)}, \tag{A6}$$

$$n_{ca} = \frac{\gamma_a(A_b + 2C)}{3A_b A_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)}, \tag{A7}$$

$$n_{cb} = \frac{\gamma_a(A_b - A_c)}{3A_b A_c + C(3A_b + 3A_c + 4\gamma_a) + \gamma_a(A_b + A_c)}. \tag{A8}$$